Graphical user interface, text

Description automatically generated

The homogeneous coordinates is  and   
=> (proven)

Graphical user interface, text

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Let and =>

Converting homogeneous coordinates to Euclidean coordinate:  
. Plugging x into both line equation of l and l’, we have:





The point x lies on both lines l and l’. Therefore, the cross product of two lines is their intersection if the lines lie on the same plane.

Graphical user interface, text

Description automatically generated

Let and =>

The equation of l becomes: 

Plugging x and x’ into l, we have:

For x:   
For x’:   
Therefore, the cross product of two points is a line that goes through them.

Graphical user interface, text

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From (c), the line that goes through points x and x’ is   
All the points lying on the line 

Converting the homogenous coordinate into Euclidean coordinate, we have:

. Plugging into the line equation above:



A screenshot of a computer

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Given a homogeneous coordinate point:   
- The translation matrix:   
- The rotation matrix:    
- The scaling matrix:   
- The Euclidean transformation matrix:   
- The similarity transformation matrix:  
  
- The affine transformation matrix incorporates both change of basis and origin:  
- The projective transformation matrix incorporates all types of transformations above:   
 

A screenshot of a computer

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Number of freedoms in  
- Translation transformation: 2 Dof  
- Euclidean transformation: 3 Dof  
- Similarity transformation: 4 Dof  
- Affine transformation: 6 Dof  
- Projection transformation: 8 Dof

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The projection or homography transformation matrix is defined up to a scale. In other words, it can be multiplied by a non zero constant without any affect on projective transformation. Let this constant be the inverse of any element in the 3x3 projection matrix. Thus, multiply the homography matrix by this constant results in one of the elements equals to 1, while the other elements are still unknown. Therefore, the projection matrix has 8 degree of freedom even though it contains 9 elements (3x3 matrix) because the number of unknowns that need to be solved for the matrix is only 8.

Graphical user interface, text, application

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We have , =>  and also  
   
or  (answer)

Graphical user interface, text, application

Description automatically generated

From (3), we have:   
Under the projective transformation, the identity is:



Therefore, this identity is invariant under projective transformation

Because projective invariants defined via homogeneous coordinates must be invariant also to arbitrary scaling of the homogeneous coordinate vectors with a non-zero scaling vector, we define a, b, c, d as the scaling factor for I identity



Therefore, two lines and two points are required to cancel out the scaling factors from the denominator and nominator. Similar construction with fewer points or lines is not possible in projective transformation because the scaling factors are not totally cancelled out and thus the identity is variant to arbitrary scaling of the homogeneous coordinate vectors.